

An Eulerian model for particle deposition under electrostatic and turbulent conditions

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Abstract

Particle deposition under the influence of electrostatic force is modeled theoretically based on the semi-empirical (three-layer) model proposed previously (J. Aerosol. Sci. 31 (2000) 463). The electrostatic forces considered are the Coulombic force and the image force. Both Boltzmann and combined diffusion and field charging mechanisms are investigated. A modified Fick's law equation accounting for Brownian and turbulent diffusion, spatially-independent external forces, i.e. gravitational and Coulombic forces, and spatially-dependent external force, i.e. image force, is presented based on the simplified three-layer model. The results show that the concentration boundary layer thickness is very thin and the previous three-layer model can be further simplified. The Coulombic force influences the particle deposition significantly while the image force is important for high charge level particles in the presence of a weak electric field. The model predictions agree very well with the literature DNS results.

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1. Introduction

Particle deposition under the influence of electrostatic force (electrophoresis) is very common in many engineering applications. Inhalation exposure, microcontamination control, aerosol filtration and separation, deposition on wafer are just a few examples (Bailey, 1997; Turner, Liguras, & Fissan, 1989; Tsai, Chang, & Lin, 1998). It has been reported in the literature that the amount of electric charges carried by airborne particles may significantly affect the deposition rate in the lung (Vincent, Johnson, Jones, & Johnson, 1981; Prodi & Mullaroni, 1985; Cohen, Xiong, Fang, & Li, 1998).

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Contaminated particle deposition on wafer has become an important issue in microelectronic industry (Forsyth & Liu, 2002).

Neutral particles must be charged before electrophoresis can occur. The amount of electric charges carried by a particle depends on the charging methods. Bipolar ions bombard the particles and the Boltzmann charge equilibrium will result. Unipolar ions, on the other hand, charge the particles through different mechanisms, diffusion or field charging. The number of charges carried by the submicrometer/supramicrometer particles through the diffusion/field charging mechanism is much greater than that carried by the Boltzmann charge equilibrium.

In the literature, for non-occupation environments, data for the charge level of airborne particles are very rare. There are sporadic results for some specific workplace environments (Jones, Adair, Mawhinney, & Gorman, 1996). Many of the surfaces of clean rooms are poor conductors, i.e. wafer surfaces, and can develop and retain a high electrostatic charge level and cause electrostatic attraction of the airborne particles (Opiolka, Schmidt, & Fissan, 1994). To the authors' best knowledge, there are no published data on the electric charges carried by bioaerosols. However, there are a few evidences showing bioaerosols may carry very high charge level (Mainelis et al., 2001).

There are many studies on particle deposition in small pipes or parallel plates taking various electrostatic external forces into account (Goo & Lee, 1996; He & Ahmadi, 1999). For the rather low flow velocity encountered indoors, their work, however, does not contribute to the understanding of turbulent transport of the submicron size range in clean rooms and in common enclosure/chamber environments. Surprisingly, the amount of research put on particle deposition in clean rooms/indoors, which is directly related to microcontamination control and human health, is far less than the amount of research on deposition in small diameter tubes/channels (Lai, 2003).

Among many Eulerian models available for (indoor) particle deposition problem, there are two important approaches to evaluate the particle eddy diffusivity, ε_p . One was first developed by Corner and Pendlebury (1951), which was modified later by Crump and Seinfeld (1981). In the original expression, Corner and Pendlebury assumed the diffusivity can be calculated by $\varepsilon_p = K_e y^m$ and they set $m = 2$, where K_e is a parameter characterizing the turbulent intensity. The exact value of the exponent m is unknown, and various values of m between 2 and 3 have been employed (refer to the review of Lai, 2003). Recently, a new semi-empirical particle deposition model (hereafter referred to the three-layer model) has been developed (Lai & Nazaroff, 2000). The loss mechanisms considered are Brownian and turbulent diffusion and gravitational effect. The model has stronger physical background than previous models, which always required ad hoc parameters estimation. Although the model was originally developed for indoor environments, it can be applied to particle deposition in pipe flow conditions (see results for details).

McMurry and Rader (1985) extended the particle deposition theory of Crump and Seinfeld (1981), by considering Brownian and turbulent diffusion, gravitational sedimentation and Coulombic drift, in an electrically charged chamber for different particle initial charge conditions. Shimada, Okuyama, Kousaka, Okuyama, and Seinfeld (1989) studied the deposition rate of singly charged particles in two very small stirred tanks (2.6 and 0.57 l). Turner and Fissan (1989) presented a theoretical model to describe particle deposition to smooth, flat surfaces from turbulently mixed gases. They examined the influence of the Coulombic force and the image force on particle deposition theoretically, and the model development was based on Crump and Seinfeld's model (1981) with $m = 2$. Many studies only focused on the drift velocity caused by Coulombic attraction. The contribution of the image force is often neglected by previous workers due to the relatively short range effect (Opiolka et al.,

1994; Schneider, Kildesø, & Breum, 1999; McMurry & Rader, 1985; Shimada et al., 1989). Unlike the Coulomb electrostatic attraction of which the mathematics required is very simple, the presence of the image force makes the governing equation highly non-linear.

For large particles, the contribution of gravitational force on deposition always overwhelms other deposition mechanisms. In order to gain more physical knowledge on individual deposition mechanisms, different orientated surfaces should be considered separately. For vertically orientated surfaces, electrophoresis may enhance the deposition rate of large particles more significantly.

The objective of the present paper is, based on the Lai and Nazaroff's previous model (2000), to develop a particle deposition model for smooth surfaces accounting for Brownian and turbulent diffusion, gravity, the Coulombic force, and the image force. The particle charging level is determined by the Boltzmann charge distribution, diffusion and field charging. The present results are focused on vertically and horizontally orientated surfaces at low air flow velocities.

2. Modeling approach

2.1. Introduction and modification of the three-layer model

The three-layer model proposed by Lai and Nazaroff (2000) considered three transport mechanisms: Brownian and turbulent diffusion and gravitational settling. A very thin particle concentration boundary layer is observed within the turbulent boundary layer, and through the concentration boundary layer the particle flux, J , is constant, which could be described by a modified form of Fick's law:

$$J = -(D + \varepsilon_p) \frac{\partial C}{\partial y} - i v_s C, \quad (1)$$

where D is the Brownian diffusion coefficient, C is the particle number concentration, y is the distance from the surface, v_s is the settling velocity, which is independent on y , and i is used to characterize the orientation of the surface, i.e. for an upward facing horizontal surface, $i = 1$; for a downward facing horizontal surface, $i = -1$; for a vertical surface, $i = 0$.

Rewriting Eq. (1) into a dimensionless form

$$v_d^+ = \left(\frac{D + \varepsilon_p}{\nu} \right) \frac{dC^+}{dy^+} + i v_s^+ C^+, \quad (2)$$

where v_d is the particle deposition velocity, defined as $v_d = |J(y=0)|/C_\infty$. The parameters are normalized by the particle concentration outside the concentration boundary layer (i.e. the bulk concentration), C_∞ , the friction velocity, u^* , and the kinematic viscosity of air, ν , as follows: $C^+ = C/C_\infty$, $y^+ = yu^*/\nu$, $v_d^+ = v_d/u^*$, $v_s^+ = v_s/u^*$. It should be noted that the friction velocity, u^* , is the only parameter to characterize the intensity of near surface turbulent flow in the model. The model assumes the particle eddy diffusivity, ε_p , and the turbulent viscosity, ν_t , are equal within the boundary layer, $\varepsilon_p = \nu_t$.

The key concept of the model is to divide the total resistance against particle deposition into three individual layers. The division is based on classical boundary layer observations (Kline, Reynolds,

Schraub, & Runstadler, 1967; Smith & Metzler, 1983). Eq. (3) are best-fitted based on the direct numerical simulation (DNS) results of Kim, Moin, and Moser (1987).

$$v_t/v = 7.669 \times 10^{-4}(y^+)^3, \quad 0 \leq y^+ \leq 4.3, \quad (3a)$$

$$v_t/v = 1.00 \times 10^{-3}(y^+)^{2.8214}, \quad 4.3 < y^+ \leq 12.5, \quad (3b)$$

$$v_t/v = 1.07 \times 10^{-2}(y^+)^{1.8895}, \quad 12.5 < y^+ \leq 30. \quad (3c)$$

Rearranging Eq. (2),

$$\left(\frac{1}{v_d^+ - iv_s^+ C^+} \right) dC^+ = \left(\frac{v}{D + \varepsilon_p} \right) dy^+. \quad (4)$$

The two boundary conditions are $C^+ = 0$ at $y^+ = r^+$, and $C^+ = 1$ at $y^+ = 30$. The boundary condition at $y^+ = r^+$ implies that the particle concentration is zero at the position where particles touch the surface, where r^+ is the normalized particle radius, $r^+ = (d_p/2)u^*/v$, and d_p is the particle diameter. The second boundary condition assumes that the particle concentration is equal to the core value ($C = C_\infty$) at the outer edge of the fluid boundary layer, $y^+ = 30$.

Substituting the first boundary condition into Eq. (4) and integrating it, we get

$$\int_0^{C^+} \left(\frac{1}{v_d^+ - iv_s^+ C^+} \right) dC^+ = \int_{r^+}^{y^+} \left(\frac{v}{D + \varepsilon_p} \right) dy^+. \quad (5)$$

By solving this equation, the dimensionless particle concentration with respect to the distance from the surface is obtained

$$C^+ = v_d^+ \frac{1 - \exp[-iv_s^+ I(y^+)]}{iv_s^+}, \quad (6)$$

where

$$I(y^+) = \int_{r^+}^{y^+} \left(\frac{v}{D + \varepsilon_p} \right) dy^+ \quad (7)$$

and the expression of $I(y^+)$ is shown in Appendix A.

The deposition velocity, v_d^+ , can be computed by substituting the second boundary condition into Eq. (5):

$$v_d^+ = \frac{iv_s^+}{1 - \exp[-iv_s^+ I(30)]}. \quad (8)$$

Through the order of magnitude analysis, the deposition rate of particles greater than $0.1 \mu\text{m}$ to downward facing horizontal surfaces is not important compared with the deposition rate to vertical and upward horizontal surfaces. Thus, in the following analysis only vertical and upward horizontal surfaces are considered. Fig. 1 demonstrates the particle concentration profiles based on Eq. (6) by setting $i = 0$ (for vertical surface) or $i = 1$ (for upward facing horizontal surface).

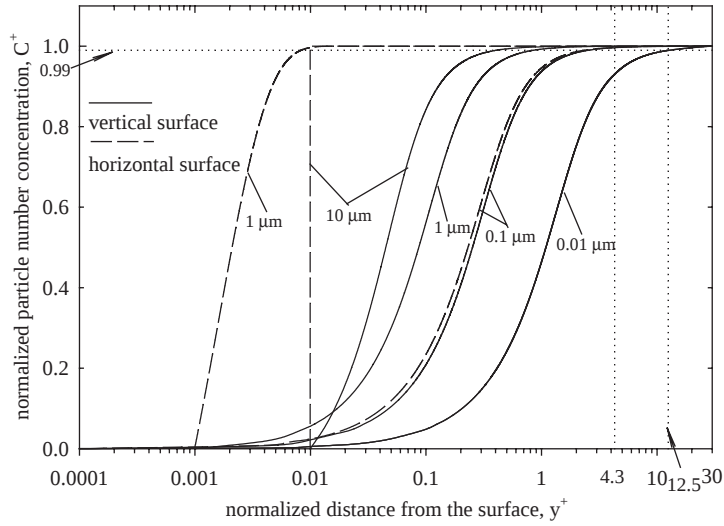


Fig. 1. Particle concentration profile within the fluid boundary layer with respect to the normalized distance from the surface. Particles of diameter $0.01 \mu\text{m}$ are insensitive to gravity and the graphs with respect to two orientations are overlapped.

The particle concentration changes rapidly in a very thin region in the vicinity of the surface where the dimensionless particle concentration rises from 0 to very close to 1. Mathematically, the concentration only reaches the core value at the infinite distance away from the wall rather than at $y^+ = 30$. Here we define the dimensionless concentration boundary layer thickness, δ^+ , as the distance at which the concentration reaches 99% of the core value. The thickness of the particle concentration boundary layer diminishes dramatically with increasing particle diameter. The particle concentration profile depends on which mechanisms controlling the particle transport within the boundary layer. Deposition of small particles is dominated by diffusion and their concentration profiles are not influenced by gravity apparently; while for large particles, the influence of gravity is very significant. For instance, for $10 \mu\text{m}$ particles, the boundary layer thickness is almost equal to the particle radius.

Following the light of the concentration profiles evaluated, the second boundary condition, $C^+ = 1$ at $y^+ = 30$, is applicable to a wide range of particle diameters. In Corner and Pendlebury's model (1951), the second boundary condition was set large enough in order to satisfy $\tan^{-1}(\delta^+) \cong \pi/2$, where $\delta^+ = \delta(K_e/D)^{1/2}$. It is convenient to set δ^+ as infinite since it will be eliminated later. However, when the image force is accounted or a set of partial differential equations need to be solved within the concentration boundary layer, an explicit value of concentration boundary layer thickness should be fixed (Shimada et al., 1989; Turner & Fissan, 1989; Turner et al., 1989).

The particle size range of interest for human respiratory health and microelectronic industry is $0.01\text{--}10 \mu\text{m}$, thus it is reasonable to neglect the contributions of outer two layers and set the second boundary condition as $C^+ = 1$ at $y^+ = 4.3$. Therefore, Eq. (8) can be modified to

$$v_d^+ = \frac{iv_s^+}{1 - \exp[-iv_s^+I(4.3)]}. \quad (9)$$

Table 1

Relative percentage error of the particle deposition velocity from the simplified three-layer model to the original three-layer model

Particle diameter (μm)	Relative percentage error for vertical surface (%)	Relative percentage error for horizontal surface (%)
0.01	7.48	7.48
0.02	2.91	2.90
0.05	0.89	0.86
0.1	0.38	0.33
0.2	0.18	0.07
0.5	0.08	0
1	0.04	0
2	0.03	0
5	0.01	0
10	0.01	0

Table 1 shows the percentage error of particle deposition velocity considering all three layers and only the first layer. For most of the engineering purposes, omitting the outer two layers causes virtually no effect on the accuracy of deposition velocity for particles larger than $0.05\ \mu\text{m}$. In addition, for smaller particles this procedure can still provide very satisfactory accuracy. With this simplification, only Eq. (3a) needs to be considered when the image force is added to the governing Eq. (1). This procedure is also useful in numerical simulations, which will save computational time significantly by considering a thinner concentration boundary layer.

The effect of an image force is always attractive, and we mainly consider the attractive effect of the Coulombic force, though the Coulombic force repels half of the charged particles at an ideal Boltzmann charge equilibrium. An attractive electrostatic force always results in a thinner boundary layer. While for the Coulombic force with repulsion, omission of the outer two layers is also valid because of following reasons: due to the low charge level, deposition of submicrometer particles is dominated by diffusion, and the Coulombic force will not change the boundary layer thickness noticeably. For supermicrometer particles, on the other hand, the Coulombic force will barely make the concentration boundary layer thickness larger than 4.3 (see Fig. 1).

2.2. Particle deposition under the influence of electrostatic force

There are four kinds of electrostatic forces acting on a charged particle near a surface: the Coulombic force (field force), the image force, the dielectrophoretic force and the dipole–dipole force. The last two forces, whose magnitudes are generally small, are usually neglected (Turner et al., 1989; He & Ahmadi, 1999; Soltani & Ahmadi, 1999). In general, the image force is also quite short range, but it may become important under some specific conditions. Coulombic force is induced when a charged particle is placed in an electric field and it can be determined by Coulomb's law. An image force is present between a charged particle of charge q and a conducting surface (corresponding to the force exerted by an image charge of $-q$ at position $-y$ from the surface, supposing that the particle is y away from the surface).

2.2.1. Coulombic force

The migration velocity of a charged particle under the influence of an electric field is

$$v_e = \frac{neE}{f}, \quad (10)$$

where n is the number of elementary charges on the particle, e is the elementary charge, $e = 1.602 \times 10^{-19}$ C, E is the y -component of the electric field and f is the drag coefficient (according to Stokes's law: $f = 3\pi\mu d_p/C_c$, μ , the dynamic viscosity of air, C_c , the Cunningham correction factor). A Coulombic force can be either toward or away from the surface, depending on the sign of charges and the direction of the electric field (electric field toward the surface is defined as positive).

Though the strength of electric field is spatially dependent, it is justifiable to assume that the electric field is constant within the extremely thin boundary layer. Consequently, the migration velocity caused by Coulombic force could be treated as another spatially independent term like the gravitational settling velocity. Combining these two mechanisms together, $v = iv_s + v_e$, where v denotes the combined migration velocity caused by spatially independent forces, i.e. the Coulombic force and gravity. In order to obtain the particle deposition with the Coulombic effect, replaces iv_s in Eq. (9) with v and the new expression is

$$v_d^+ = \frac{v^+}{1 - \exp[-v^+I(4.3)]}, \quad (11)$$

where $v^+ = v/u^*$. In the following discussion, the combined migration velocity, v , will be used instead of iv_s .

2.2.2. Image force

The expression of the drift velocity caused by image force is

$$v_i = \frac{K_E n^2 e^2}{4f} \Phi(\varepsilon) \frac{1}{y^2}, \quad (12)$$

where K_E is the electrostatic constant of proportionality (SI units), $K_E = 9.0 \times 10^9$ N m²/C². The term $\Phi(\varepsilon)$ is the dielectric constant factor, $\Phi(\varepsilon) = (\varepsilon_2 - \varepsilon_1)/(\varepsilon_2 + \varepsilon_1)$, where ε_1 , ε_2 are the dielectric constants of gas and surface material, respectively. For dry air, $\varepsilon_1 \cong 1$. For perfect insulators, $\varepsilon_2 = 1$ and $\Phi(\varepsilon) = 0$, the image force vanishes; for perfect conductors, $\varepsilon_2 = \infty$ and $\Phi(\varepsilon) = 1$, the image force is maximum. The sign of charges is not important because the effect of an image force is always attractive. Considering the migration velocity by image force is dependent on y , the migration velocity at unit distance away from the surface, $v_{i,1}$, can be defined to simplify the expression. Let $v_i = v_{i,1}/y^2$.

Adding the drift velocity expression to the right-hand side of the governing Eq. (2), it becomes

$$v_d^+ = \left(\frac{D + \varepsilon_p}{v} \right) \frac{dC^+}{dy^+} + v^+ C^+ + \frac{v_{i,1}^+}{y^{+2}} C^+, \quad (13)$$

where $v_{i,1}^+ = v_{i,1} u^*/v^2$. Rearranging Eq. (13)

$$\frac{dC^+}{dy^+} + \left(v^+ + \frac{v_{i,1}^+}{y^{+2}} \right) \frac{v}{D + \varepsilon_p} C^+ = \frac{v}{D + \varepsilon_p} v_d^+. \quad (14)$$

The solution of Eq. (14) is as follows (Kreyszig, 1999):

$$C^+ = \frac{\int_{r^+}^{y^+} M(y^+) r(y^+) dy^+ + A}{M(y^+)}, \quad (15)$$

where

$$M(y^+) = \exp \left(\int_{r^+}^{y^+} p(y^+) dy^+ \right), \quad (16a)$$

$$p(y^+) = \left(v^+ + \frac{v_{i,1}^+}{y^{+2}} \right) \frac{v}{D + \varepsilon_p}, \quad (16b)$$

$$r(y^+) = \frac{v}{D + \varepsilon_p} v_d^+ \quad (16c)$$

and A is a constant, which can be obtained by the boundary conditions: $C^+ = 0$ at $y^+ = r^+$, and $C^+ = 1$ at $y^+ = 4.3$. Substituting the boundary conditions, we get $A = 0$ and

$$M(4.3) = \int_{r^+}^{4.3} M(y^+) r(y^+) dy^+. \quad (17)$$

Considering $r(y^+) = v_d^+ \cdot v/(D + \varepsilon_p)$ and v_d^+ is independent on y^+ , the general solution with gravitational, Coulombic and image forces can be obtained as follows:

$$v_d^+ = \frac{M(4.3)}{\int_{r^+}^{4.3} M(y^+) \frac{v}{D + \varepsilon_p} dy^+}. \quad (18)$$

Eq. (18) cannot be integrated analytically because of the complexity of the exponent function involved and it must be integrated numerically. Prior to the numerical integration, $M(y^+)$ should be solved analytically. The expression of $M(y^+)$ is attached in Appendix B. In case of no image force, it can be shown that Eq. (18) is equivalent to Eq. (11).

2.3. Particle charge distribution

Most aerosol particles carry some electric charges and the charge levels depend on the charging mechanisms. In order to examine the effects of the Coulombic force, the image force and these two combined together, two frequently encountered charging mechanisms, Boltzmann charge equilibrium and combined diffusion and field charging mechanism, are engaged in the following analysis.

Particles in a bipolar ionic atmosphere will tend to attain an equilibrium charge state called the Boltzmann equilibrium. The fraction of particles $f(n)$ of a given diameter d_p having n elementary

Table 2

Neutral fraction, average absolute charge and rms charge of various particle diameters at the Boltzmann equilibrium

Particle diameter (μm)	Boltzmann equilibrium			Diffusion and field charging		
	Neutral fraction (%)	Average charge	RMS charge	Diffusion charging	Field charging	Combined
0.01	99.3	0.007	0.085	0.276	0.0007	0.276
0.02	89.3	0.107	0.328	0.671	0.0028	0.673
0.05	59.8	0.415	0.665	2.08	0.0174	2.095
0.1	42.3	0.677	0.943	4.77	0.0694	4.84
0.2	29.9	1.01	1.33	11	0.277	11
0.5	18.9	1.65	2.11	31	1.74	33
1	13.4	2.36	2.98	68	6.94	75
2	9.46	3.35	4.22	149	28	176
5	5.98	5.31	6.67	412	174	586
10	4.23	7.52	9.43	886	694	1580

The number of charges acquired by diffusion, field and combined charging mechanisms of particles at the saturation charge level is also presented.

charges (either positive or negative) is given by (Hinds, 1999):

$$f(n) = \frac{\exp(-K_E n^2 e^2 / d_p k T)}{\sum_{n=-\infty}^{\infty} \exp(-K_E n^2 e^2 / d_p k T)}, \quad (19)$$

where k is the Boltzmann constant, $k = 1.3806 \times 10^{-23}$ J/K, and T is the absolute temperature. The average absolute charge number, n_{ave} , and the root mean square (rms) charge per particle, n_{rms} , are given as

$$n_{\text{ave}} = \sum_{n=0}^{\infty} |n| f(|n|), \quad (20a)$$

$$n_{\text{rms}} = \left[\sum_{n=-\infty}^{\infty} n^2 f(n) \right]^{1/2}. \quad (20b)$$

By the charge distribution function, the results of the uncharged fraction, the mean absolute charge and the rms charge number with respect to different particle diameters are listed in Table 2. For particles larger than 0.2 μm , the empirical approximation for average number of charges, $n_{\text{ave}} \cong 2.37\sqrt{d_p}$, where d_p is in μm , could be used with sufficient accuracy (Hinds, 1999). Another empirical equation for the rms charge, $n_{\text{rms}} \cong 2.9\sqrt{d_p}$ (d_p is also in μm), was used by Turner and Fissan (1989). However, both of these empirical expressions overestimate the charge level of small particles; hence, the exact charge distribution function is used in present model instead of these empirical expressions. The charge level of particles at the Boltzmann charge equilibrium is relatively low compared with other charging mechanisms. In the present work, the upper bound charge number carried by particles is assumed to be as high as 100 times of the average absolute charge or rms charge. Although in reality only a few percentages of particles could reach such a high charge level, it is still possible that under certain charging conditions the particles can carry such a charge level and it is useful in evaluating the impact of high charge levels.

Particles mixed with unipolar ions are charged by random collisions between the ions and the particles due to the Brownian motion. For particles less than 1 μm , diffusion charging is the main charging mechanism. The approximate number of charges n acquired by a particle of diameter d_p by diffusion charging during a time t is (Hinds, 1999)

$$n = \frac{d_p k T}{2 K_E e^2} \ln \left(1 + \frac{\pi K_E d_p \bar{c}_i e^2 N_i t}{2 k T} \right), \quad (21)$$

where $\bar{c}_i$ is the mean thermal speed of the ions ($\bar{c}_i = 240$ m/s) and N_i is the ion concentration.

In the presence of a strong electric field, particles are charged by frequent collisions with unipolar ions in rapid motion. This is referred to the field charging. The effect of field charging is more significant for particles larger than 1 μm and the charge magnitude increases with the square of particle diameter. After sufficient time at a given charging condition, the maximum number of charges n acquired by one particle is (Hinds, 1999)

$$n = \left(\frac{3\varepsilon_0}{\varepsilon_0 + 2} \right) \left(\frac{E d_p^2}{4 K_E e} \right), \quad (22)$$

where ε_0 is the dielectric constant of the particle material. Particles reaching the maximum charge number at combined diffusion and field charging are said to be at saturation charge.

The number of charges acquired by particles at saturation charge condition is listed in Table 2.

3. Results and discussion

Referring to Eq. (10), the drift velocity caused by Coulombic force is dependent on the product nE . Since the particle charge level is a function of particle diameter, $\varphi = n/n_{\text{ave}}$ is defined to characterize different charge levels. Hence, the Coulombic force can be described by the product φE . A surface that induces an electric field strength in the order of 10^3 V/m will result in a product φE of the order of 10^5 V/m, by assuming that φ could reach a value of 100. Fig. 2 (for vertical surface) and Fig. 3 (for upward facing horizontal surface) show the influence of the Coulombic force on the deposition velocity for two turbulent levels, $u^* = 3$ and 10 cm/s, to a perfect insulator surface (i.e. no image force). The Coulombic force enhances deposition rate significantly especially for small turbulent intensities. The Coulombic force becomes the dominant mechanism for $\varphi E \geq 10^4$ and the deposition velocity is dependent solely on φE , regardless of the turbulent intensities. That means that in the presence of a strong electric field, the turbulent intensity variation will not have any significant effects on particle deposition. Because there are fewer charges carried by particles smaller than 0.02 μm , the influence of the Coulombic force is reduced in this region. For a horizontal surface, gravitational sedimentation dominates the deposition of particles larger than 0.2 μm , but for large φE , the contribution of the Coulombic force for larger particles cannot be ignored.

The migration velocity caused by image force is dependent on the product $n^2 \Phi(\varepsilon)$ as shown in Eq. (12). Since the image force is always attractive and its magnitude is proportional to the square of the charge on the particle, a parameter $\phi = n/n_{\text{rms}}$ is defined to characterize the charge level based on the rms charge of Boltzmann charge distribution and the image force can be described by the product $\phi \sqrt{\Phi(\varepsilon)}$ (Peters, Cooper, & Miller, 1989). It is found that, for particles at the Boltzmann rms value, the enhancement due to the image force is very small (results are not shown). Therefore,

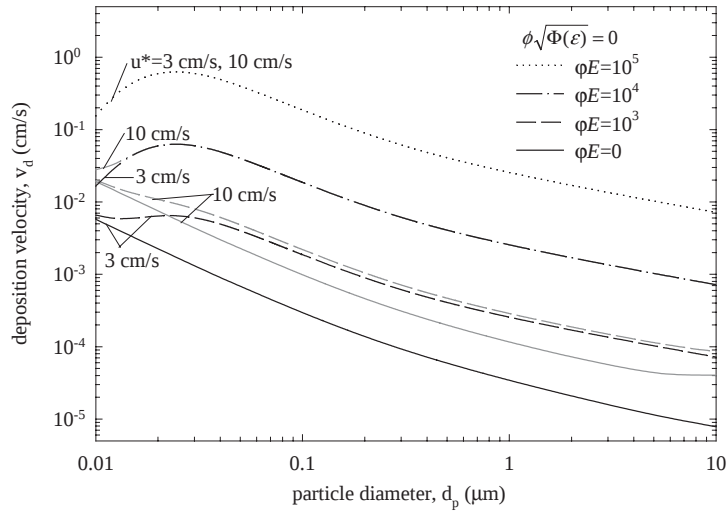


Fig. 2. Particle deposition velocity as a function of particle size, turbulent intensity and Coulombic force strength for a vertically orientated surface. For $\phi E = 10^4$ and 10^5 , the graphs with respect to two turbulent intensities are overlapped.

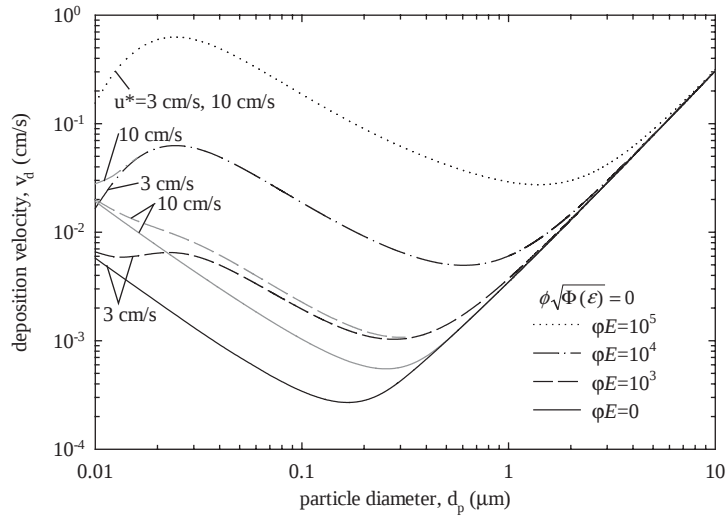


Fig. 3. Particle deposition velocity as a function of particle size, turbulent intensity and Coulombic force strength for an upward facing horizontal surface. For $\phi E = 10^4$ and 10^5 , the graphs with respect to two turbulent intensities are overlapped.

two typical values, $\phi\sqrt{\Phi(\varepsilon)} = 10$ and 100 , are assumed and used to demonstrate the effect of the image force and the results are shown in Fig. 4 (for vertical surface) and Fig. 5 (for upward facing horizontal surface). It is assumed here that the electric field strength is zero (i.e. no Coulombic force). It is clearly shown that the influence of the image force is proportional to the particle charge level. Unlike the case of Coulombic force where the friction velocity has insignificant influence on the deposition, the deposition velocity increases with u^* and $\phi\sqrt{\Phi(\varepsilon)}$. For a horizontal surface, the

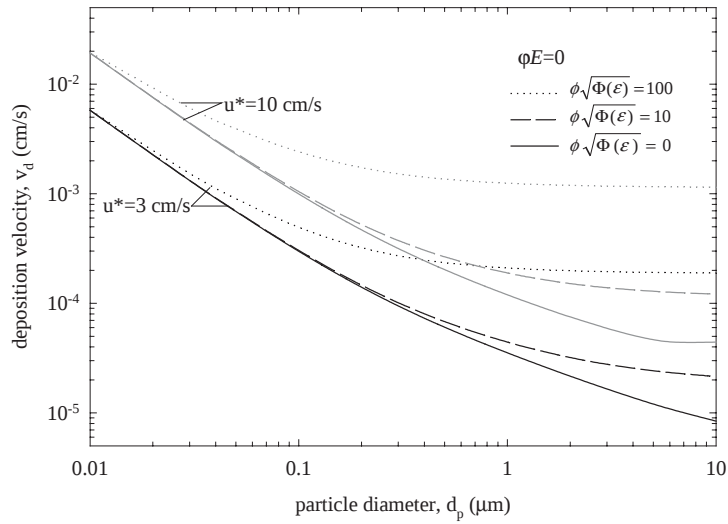


Fig. 4. Particle deposition velocity as a function of particle size, turbulent intensity and image force strength for a vertically orientated surface.

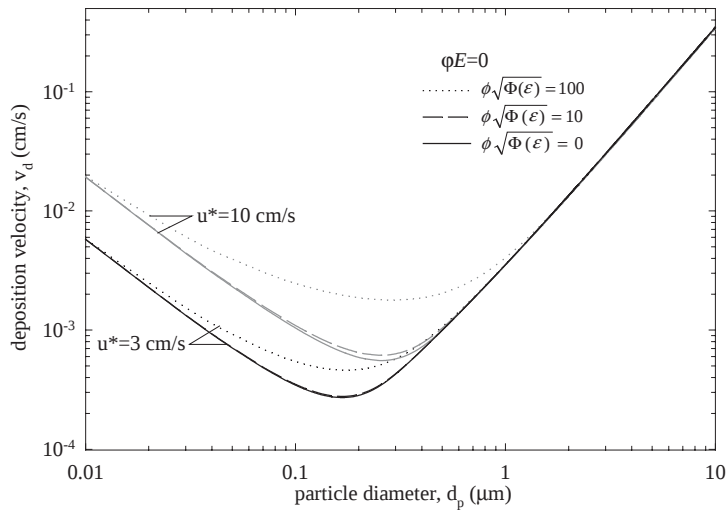


Fig. 5. Particle deposition velocity as a function of particle size, turbulent intensity and image force strength for an upward facing horizontal surface.

image force only influences the deposition of accumulation mode particles. It should be noted that for particle with 10 times of Boltzmann rms charge, the deposition enhancement caused by image force can be neglected for a horizontal surface.

Recently, several direct numerical simulation (DNS) results on charged particle deposition in turbulent channel flow were reported (Soltani, Ahmadi, Ounis, & McLaughlin, 1998; Soltani & Ahmadi, 1999). The channel was vertically oriented with constant strong electric field. Particles were at the Boltzmann charge equilibrium or saturation charge and the electrostatic forces considered were

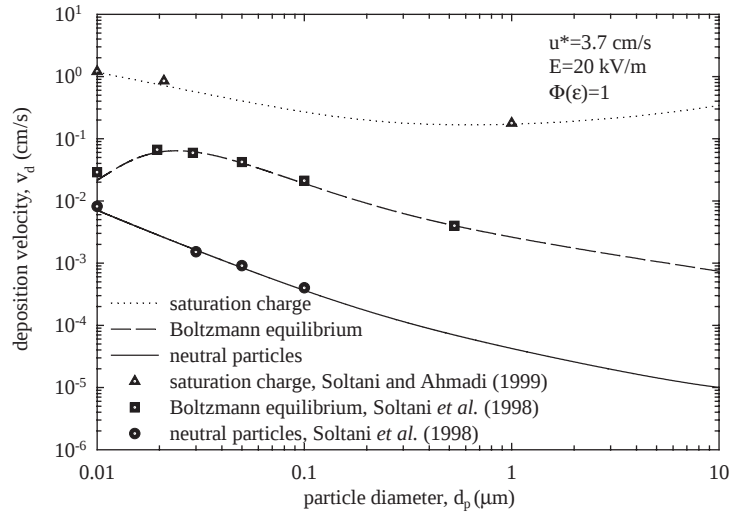


Fig. 6. Comparison of DNS results (Soltani et al., 1998; Soltani & Ahmadi, 1999) with predictions from the current model for a vertically orientated surface as a function of particle size and two charge levels.

Coulombic and image forces. The friction velocity was set to 3.7 cm/s, and the authors highlighted that this is a typical value for clean room environments. A comparison between the results of current model and DNS was made and the results are demonstrated in Fig. 6. For particles at the Boltzmann charge equilibrium, particles are assumed to carry random integer number of positive or negative charges in accord with the Boltzmann distribution. The deposition velocity of particles at the Boltzmann charge equilibrium, $v_{d,B}$, is obtained as

$$v_{d,B} = \sum_{n=-\infty}^{\infty} v_{d,n} f(n), \quad (23)$$

where $v_{d,n}$ is the deposition velocity of particles with n elementary charges, which can be calculated from Eq. (19). Very good agreement was observed between the current model and the DNS results (Soltani et al., 1998). Besides the Boltzmann distribution, Soltani and Ahmadi (1999) utilized the same DNS simulation methodology to study particle deposition at the saturation charge, it can be seen that excellent agreement between the present work and their DNS results. If deposition velocity is the only concern, the result obtained by the present work shows very high accuracy comparable to DNS; however, the computation resources required is negligible to that required by DNS. Good agreement with experimental data conducted in a spherical chamber (Cheng, 1997) was also observed for the previous three-layer model. Together with the present results, there are some strong evidences that the model works very well for both indoor and pipe flow conditions.

4. Conclusions

In this paper, particle deposition onto vertical and upward facing horizontal surfaces under the influence of electrostatic force is studied theoretically. For particles larger than 0.01 μm , the particle

concentration boundary layer is within the first layer, $y^+ = 4.3$. The second boundary condition is modified to $C^+ = 1$ at $y^+ = 4.3$ and satisfactory accuracy is obtained. From the results obtained, for particle deposition studies, one can put the particle concentration uniformly up to $y^+ = 4.3$. It will reduce computer resources significantly for CFD simulations of complex geometries. For the current work, the numerical scheme required to solve the image force governing equation is also simplified notably.

The influences of the Coulombic force, the image force and these two combined together are examined based on the Boltzmann equilibrium and the combined diffusion and field charging. The particle deposition velocity depends on particle size and charge level, turbulent intensity, electric field strength, surface material and orientation. The Coulombic force plays an important role in particle deposition. For particles at extremely high charge level and in the presence of a strong electric field, the Coulombic force contributes significantly by enhancing deposition of accommodation mode particle size. The image force is important when the particle charge level is high and the electric field strength is weak. In many applications, the effect of the image force can be negligible. Predictions of this model agree very well with the DNS results obtained in a vertical channel with low turbulent intensity. Apparently, the three-layer model works for both indoor and pipe flow environments and further experimental validations are required to draw more solid conclusions.

Appendix A

The expression of $I(y^+)$ is obtained by substituting Eqs. (3) into Eq. (7). In order to get an analytical solution, the integral for the outer two layers ($y^+ \geq 4.3$) is simplified by assuming that Brownian diffusivity can be ignored relative to the much larger turbulent diffusivity ($D \ll \varepsilon_p$).

When $r^+ \leq y^+ \leq 4.3$:

$$I(y^+) = 3.64 Sc^{2/3}(a - b), \quad (\text{A.1-1})$$

where a and b are as follows:

$$a = \frac{1}{2} \ln \left[\frac{(10.92 Sc^{-1/3} + y^+)^3}{Sc^{-1} + 0.0609} \right] + \sqrt{3} \tan^{-1} \left[\frac{2y^+ - 10.92 Sc^{-1/3}}{\sqrt{3} 10.92 Sc^{-1/3}} \right], \quad (\text{A.1-2a})$$

$$b = \frac{1}{2} \ln \left[\frac{(10.92 Sc^{-1/3} + r^+)^3}{Sc^{-1} + 7.669 \times 10^{-4}(r^+)^3} \right] + \sqrt{3} \tan^{-1} \left[\frac{2r^+ - 10.92 Sc^{-1/3}}{\sqrt{3} 10.92 Sc^{-1/3}} \right]; \quad (\text{A.1-2b})$$

when $4.3 < y^+ \leq 12.5$:

$$I(y^+) = I(4.3) + 38.55 - 549(y^+)^{-1.821}; \quad (\text{A.1-3})$$

when $12.5 < y^+ \leq 30$:

$$I(y^+) = I(4.3) + 44.15 - 105(y^+)^{-0.889}. \quad (\text{A.1-4})$$

Here, Sc is the Schmidt number, $Sc = \nu/D$.

Appendix B

The analytical expression of $M(y^+)$ is obtained by substituting Eqs. (3a) and (16b) into Eq. (16a) and integrating it.

$$M(y^+) = \exp \{v^+ V(y^+) + v_{i,1}^+ VI(y^+)\}, \quad (\text{A.2-1})$$

$$V(y^+) = 3.642 Sc^{2/3} \left[\frac{1}{2} \ln \left(\frac{(0.092 y^+ + Sc^{1/3})^3}{7.669 \times 10^{-4} y^{+3} + Sc^{-1}} \right) + \sqrt{3} \tan^{-1} \left(\frac{2y^+ - 10.92 Sc^{-1/3}}{\sqrt{3} \times 10.92 Sc^{-1/3}} \right) \right], \quad (\text{A.2-2})$$

$$VI(y^+) = -\frac{Sc}{y^+} + 0.031 Sc^{4/3} \left[\frac{1}{2} \ln \left(\frac{(0.092 y^+ + Sc^{1/3})^3}{7.669 \times 10^{-4} y^{+3} + Sc^{-1}} \right) + \sqrt{3} \tan^{-1} \left(\frac{2y^+ - 10.92 Sc^{-1/3}}{\sqrt{3} \times 10.92 Sc^{-1/3}} \right) \right]. \quad (\text{A.2-3})$$

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